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“Anomalous” $U(1)$ Symmetry in Orbifold String Models

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Abstract

“Anomalous” $U(1)$ gauge symmetry with Green-Schwarz anomaly cancellation mechanism is discussed in the orbifold construction of four-dimensional heterotic string models. Some conditions are given as criteria to have “anomalous” $U(1)$ in orbifold string models. In particular, “anomalous” $U(1)$ is absent if the massless twisted matter has no mixing between visible and hidden sectors or if a certain type of discrete symmetries are found. We then give a general procedure for classifying orbifold models with “anomalous” $U(1)$ and for identifying the “anomalous” $U(1)$ basis. We illustrate our procedure in Z_3 and Z_4 orbifold models. According to our procedure, the classification of “anomalous” $U(1)$ can be reduced to the classification in the absence of a Wilson line. We also discuss discrete symmetries left unbroken after the “anomalous” $U(1)$ breaking. This includes a possible relation between “anomalous” $U(1)$ and discrete R -symmetries.

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1 Introduction

Superstring theory is a promising candidate of the unified theory. Lacking for the dynamical principle determining the true string vacuum, many efforts have been devoted to construct semi-realistic string models directly in four dimensions. The physical content turns out to be rich enough that there have been found many semi-realistic models. It is important to extract such model-independent properties characteristic to string models that are not shared by usual field theoretic approach to unified theories.

The original surprise in superstring theory was the anomaly cancellation found in ten dimensions by Green and Schwarz[1]. As the result of the global consistency condition, modular invariance, of the world-sheet theory, we have many examples of four-dimensional string models which show a miraculous pattern of gauge anomaly cancellation. In particular, we often have string models possessing so-called “anomalous” $U(1)$ gauge symmetry. This $U(1)$ gauge symmetry has non-vanishing contributions to anomalies from chiral fermions. In fact, these anomalies are canceled via four-dimensional counterpart of Green-Schwarz mechanism, *i.e.*, by assigning the nonlinear transformation to the axion-like field. Since such anomaly cancellation mechanism of anomalous $U(1)$ is intimately related to the consistency of string theory, we can expect characteristic and interesting consequences from this mechanism. [A discussion related to this point can be found in Ref. [2], where an anomalous $U(1)$ is used to derive a constraint on non-perturbative effects in string theory.]

Many string models are known which possess the anomalous $U(1)$ gauge symmetry. These, however, have been analyzed in a model-dependent way and no criteria for the appearance of the anomalous $U(1)$ gauge symmetry has been completely clarified yet. Which class of string models does give rise to the anomalous $U(1)$? At present, we need a detailed analysis of massless spectrum in each model and struggle with $U(1)$ charges to see whether a string model contains an anomalous $U(1)$. This situation is quite unsatisfactory not only for theoretical purpose but also for phenomenological applications that we mention shortly. Therefore a systematic analysis is desirable. To do this is the main motivation of the present work.

The anomalous $U(1)$ gauge symmetry with Green-Schwarz anomaly cancellation mechanism is very interesting by itself and has actually received renewed interests. The presence of mixed $U(1)$ -gravitational anomaly in particular implies that its $U(1)$ generator Q is not traceless $\text{Tr } Q \neq 0$ leading to the generation of Fayet-Iliopoulos term[3]. This in turn breaks¹ the anomalous $U(1)$ automatically[4]. The breaking scale is calculated[5] to be just below the string

¹ Otherwise, the supersymmetry itself breaks down. This possibility is potentially interesting if we consider the “dual” theory in which the supersymmetry breaking would show up in a non-perturbative manner.

scale:

$$M_A^2 = \frac{\text{Tr } Q}{192\pi^2} M_{\text{st}}^2 . \quad (1)$$

Unlike the conventional scheme of grand unifications, we need not entangle with complicated Higgs structure. Many interesting applications of anomalous $U(1)$ come from this automatic breaking.

There are several contexts in which applications of the anomalous $U(1)$ are addressed. The anomalous $U(1)$ has most widely been discussed in constructing a string model with a realistic gauge group and matter content; the anomalous $U(1)$ breaking sometimes triggers further breaking of gauge symmetries and therefore provides a way of reducing the rank of gauge group[6, 7, 8]. It was also pointed out[9] that we have an interesting way of calculating the weak mixing angle without any grand unification symmetries since the anomaly cancellation condition relates the normalization of gauge couplings to anomaly coefficients. Also as several authors observed, the breaking scale M_A is quite impressive for the origin of various hierarchical structures in particle physics. Utilizing the ratio M_A/M_{st} , higher dimensional couplings could explain hierarchical structures in the fermion masses and mixing angles. Actually there are several proposals[10, 11, 12] for realistic fermion mass matrices based on a family $U(1)$ symmetry, which is often anomalous, while stringy selection rules were used in Ref. [13]. Furthermore, generalizing $U(1)$ D -term contributions in supergravity models[14, 15], it was argued[16, 17, 18] that the anomalous $U(1)$ gives a new source to non-universality of soft breaking scalar masses and the cosmological constant through Fayet-Iliopoulos D -term, both of which should be taken into account when we are to have the universal scalar mass and vanishing cosmological constant.

A remarkable possibility has recently been pointed out on the basis of these developments: An anomalous $U(1)$ can be used to construct a model with supersymmetry breaking[19, 20]. Also in cosmological context, some authors have argued[21, 22] that it can play a role in constructing a model of inflation. A possible solution to doublet-triplet splitting problem has also been suggested[23].

Now, it is desirable to take a systematic approach to string models with anomalous $U(1)$ in order to further explore these possibilities. The first issue to be discussed is to clarify the condition under which we have an anomalous $U(1)$ gauge symmetry in string models. Then the second issue is the pattern of the anomalous $U(1)$ breaking: to characterize the flat directions along which the anomalous $U(1)$ breaking occurs and to see what kind of consequences we have after such breaking.

In this paper, we shall mainly discuss the first issue by examining orbifold string models. We find that the appearance of anomalous $U(1)$ is constrained by several reasons. We show in particular that an orbifold model possesses an anomalous $U(1)$ only if it contains massless twisted matter which leads to the mixing between the visible and hidden sectors. We also give several examples of discrete symmetries that forbid the appearance of anomalous $U(1)$. Moreover,

we argue that the analysis of orbifold models in a Wilson line background can essentially be reduced to the analysis in the absence of a Wilson line. We then give a general procedure for classifying orbifold string models which possess anomalous $U(1)$.

Concerning the second issue, we give a brief discussion on discrete symmetries unbroken after the anomalous $U(1)$ breaking. These discrete symmetries would be relevant to phenomenological problems such as suppression of dangerous couplings. We also suggest a possible relation of the anomalous $U(1)$ to discrete R symmetries.

Our interest on the discrete R symmetry originates not only from phenomenological but also string theoretical aspects. It was argued[24, 25] that a certain discrete R symmetry can protect (0,2) string vacua against the instability[26] due to the world-sheet instanton effects. In particular, Dine and Seiberg constructed an example of such R symmetry in a (2,2) string vacuum (the standard embedding of Z_3 orbifold) which guarantees the existence of the flat directions along which the (2,2) vacuum can be deformed into (0,2) ones. Other aspects concerning the stability of (0,2) vacua are recently discussed in Refs. [27, 28, 29]. This issue is beyond the scope of this paper.

This paper is organized as follows. In the next section, we give a review of the orbifold construction. In particular we recall the expressions for the mass formula and generalized GSO projection operator which determine massless spectrum. We also recall the formulas for the $U(1)$ level and charges. In section three, after recapitulating the universal nature of $U(1)$ anomalies in string theories, we discuss the conditions for a $U(1)$ to be anomalous. Our procedure for classifying and identifying the anomalous $U(1)$ is presented in section four. We illustrate it by working out Z_3 orbifold models. [A classification of Z_4 orbifold models is given in Appendix B.] In section five, we briefly discuss the issues on the flat directions and anomalous $U(1)$ breaking. The final section is devoted to concluding remarks. Some concrete examples for the orbifold models with the anomalous $U(1)$ are given in appendices.

2 Orbifold Construction

In this section, we give a review of the orbifold construction of four-dimensional string models[30]. For details, see Refs. [31, 32, 33].

The Hilbert space of heterotic string theory is a tensor product of the right-moving sector, which is responsible for space-time supersymmetry, and the left-moving sector, which gives rise to gauge symmetries. The right moving sector is a superconformal field theory of the (4+6)-dimensional string coordinates $X^{\mu=0,1,2,3}$ and $X^{i=1,2,3}$ (and their complex conjugates $X^{\bar{i}}$) as well as their world-sheet superpartners, NSR fermions $(\psi^i, \psi^{\bar{i}}; \psi^\mu)$. The latter are conveniently bosonized, $\psi^t = -ie^{iH^t}$, into H^t ($t = 1, 2, 3$ and $t = 4, 5$ correspond to six- and four-

dimensional parts, respectively) which are related to Cartan part of $SO(9,1)$ current algebra. The momenta p^t of H^t span an $SO(9,1)$ weight lattice $\Gamma_{SO(9,1)}$. Neveu-Schwarz sector (space-time boson) corresponds to vectorial weights p_v^t with integer entries and Ramond sector (space-time fermion) to spinorial weights p_s^t with half-integer entries.

The conformal field theory of the left moving sector includes four-dimensional string coordinates \bar{X}^μ , and six-dimensional parts \bar{X}^i and $\bar{X}^{\dot{i}}$. Adopting the bosonic formulation for the gauge sector, we also have anti-chiral coordinates parametrizing the maximal torus of $E_8 \times E'_8$, $\bar{X}^{\dot{I}} = (\bar{X}^I, \bar{X}^{I'})$, whose momenta $P^{\dot{I}} = (P^I, P^{I'})$ span an $E_8 \times E'_8$ root lattice $\Gamma_{E_8 \times E'_8}$ ($I, I' = 1, \dots, 8$). The root vectors of E_8 satisfy $(P^I)^2 = 2$ and are represented as

$$P^I = (\pm 1, \pm 1, 0, 0, 0, 0, 0, 0) \equiv (\pm 1, \pm 1, 0^6) , \quad (2)$$

$$P^I = \left(\pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2} \right) \quad (3)$$

with the even number of minus signs for the latter case. Here and hereafter the underline means permutations and repeated entries are indicated by superscripts.

In a toroidal compactification to four dimensions, the resultant space-time supersymmetry charges form four-dimensional representation of $SU(4) \simeq SO(6)$ subgroup of $SO(9,1)$. In the orbifold construction, we divide the six-dimensional torus T^6 by its discrete isometry, called point group P , in order to have precisely $N = 1$ supersymmetry. We choose the $SO(9,1)$ weight p_Q^t of the surviving supercharge to be $p_Q^t = \pm(1, 1, 1; \pm 1, \pm 1)/2$ (with even $-$'s).

We concentrate on symmetric Z_N orbifolds. Then the six-dimensional part of the string coordinates combines into $X^i(z, \bar{z}) = X^i(z) + \bar{X}^i(\bar{z})$ which become the coordinates of a torus $T^6 = \mathbf{R}^6/\Lambda$ through dividing by a root lattice Λ of a semi-simple Lie algebra. The orbifold modding is realized through a further division of this torus by the point group $P = Z_N$, which is generated by a twist θ , $\theta^N = 1$. These operations can equivalently be expressed through division by a space group $S = (\Lambda, P)$; $T^6/P \simeq \mathbf{R}^6/S$.

In order to achieve the modular invariance, we should include twisted sectors. In Z_N orbifold models, we have the k -th twisted sector T_k ($k = 1, 2, \dots, N-1$) as well as the untwisted sector U . In the T_k -sector, the six-dimensional coordinates satisfy

$$X^i(e^{2\pi i} z, e^{-2\pi i} \bar{z}) = e^{2\pi i k v_i} X^i(z, \bar{z}) + e^i , \quad (4)$$

where $e^{2\pi i v_i}$ is the eigenvalue of the twist θ on X^i and the vector e^i belongs to the defining lattice Λ of the torus T^6 . The right-moving world-sheet supersymmetry then requires

$$H^i(e^{2\pi i} z) = H^i(z) + 2\pi k v^i . \quad (5)$$

The shift vector $v^t = (v^i; 0, 0)$ in $\Gamma_{SO(9,1)}$ should be orthogonal to the weight p_Q^t of the surviving supercharge. For the Abelian embedding of the twist into

gauge group, we associate a shift $V^{\hat{I}}$ to the rotation θ as well as a Wilson line $a^{\hat{I}}$ to the translation e^i of the space group. Then the boundary condition of sixteen-dimensional gauge coordinates is

$$\bar{X}^{\hat{I}}(e^{-2\pi i}\bar{z}) = \bar{X}^{\hat{I}}(\bar{z}) + 2\pi k V^{\hat{I}} + 2\pi m^i a_i^{\hat{I}}, \quad (6)$$

where the integer m^i labels fixed points. The modular invariance restricts the possible choice of shift vectors in $\Gamma_{E_8 \times E'_8}$ and Wilson lines according to

$$N \left[\sum_{i=1}^3 (v^i)^2 - \sum_{\hat{I}=1}^{16} (V^{\hat{I}} + m^i a_i^{\hat{I}})^2 \right] = \text{even integer} . \quad (7)$$

All the possible shifts $v^t = (v^i; 0, 0)$ and $V^{\hat{I}} = (V^I; V^{I'})$ are known for each Z_N orbifold construction[34]. The simplest choice is the standard embedding; $V^{\hat{I}} = (v^i, 0^5; 0^8)$, $a^{\hat{I}} = 0$. We also refer to the following type of the shift as a *quasi-standard embedding*

$$V^{\hat{I}} = (v^i, 0^5; V^{I'}) . \quad (8)$$

On-shell string states are created by vertex operators acting on the vacua of (super)conformal field theories on the string world sheet, $V_R(z) |0\rangle_R \otimes V_L(\bar{z}) |0\rangle_L$. The internal part of the vertex operators takes the form

$$V_R \sim e^{ip_R^i X^i} e^{ip^t H^t}, \quad V_L \sim e^{ip_L^i \bar{X}^i} e^{iP^{\hat{I}} \bar{X}^{\hat{I}}} . \quad (9)$$

In twisted sectors, we should drop the momenta $p_{R,L}^i$ and replace the $SO(9, 1)$ and $E_8 \times E_8$ momenta with shifted ones defined by

$$\tilde{p}^t \equiv p^t + k v^t, \quad \tilde{P}^{\hat{I}} \equiv P^{\hat{I}} + k V^{\hat{I}} + m^i a_i^{\hat{I}} . \quad (10)$$

Each twisted sector T_k has several subsectors corresponding to fixed points labeled by the twist θ^k and e^i (by the conjugacy class of the space group, to be precise). The vertex operator for such twisted state includes the twist field [35], $\sigma(k, e)$, which creates the twisted vacuum $|k, e\rangle = \sigma(k, e) |0\rangle$ and expresses the twisted boundary condition (4) of the internal string coordinates $X^{i=1,2,3}$. These twist fields contribute to the conformal dimension of the ground state in the k -th twisted sector by an amount

$$c^{(k)} \equiv \frac{1}{2} \sum_{i=1}^3 \eta_{(k)}^i (1 - \eta_{(k)}^i), \quad \eta_{(k)}^i \equiv |k v^i| - \text{Int} |k v^i| . \quad (11)$$

We also recall that there arise some complications concerning the twisted vacua for higher twisted sectors ($k = 2, \dots, N-2$) in non-prime order orbifolds ($N = 4, 6, 8, 12$), which are relevant for the generalized GSO projection. In this case, the fixed points of the higher twist θ^k do not necessarily fixed by the single twist θ

but transform into each other. Therefore we have to take their linear combination to form an eigenstate under the single twist[36]. In Ref. [33], such eigenstates were explicitly constructed with their eigenvalues $e^{i\gamma}$ under the single twist.

Mass formulas and physical states are most easily described in light-cone gauge.² Mass formulas are obtained by counting the conformal dimensions of vertex operators and are given, for the right- and left-moving T_k -sectors (untwisted sector U for $k = 0$), respectively, by

$$\frac{1}{8}m_R^2 = \frac{1}{2} \sum_{i=1}^3 (p_R^i)^2 + \frac{1}{2} \sum_{t=1}^4 (\tilde{p}^t)^2 + N_R^{(k)} - \frac{1}{2} + c^{(k)} , \quad (12)$$

$$\frac{1}{8}m_L^2 = \frac{1}{2} \sum_{i=1}^3 (p_L^i)^2 + \frac{1}{2} \sum_{\hat{I}=1}^{16} (\tilde{P}^{\hat{I}})^2 + N_L^{(k)} - 1 + c^{(k)} , \quad (13)$$

where oscillator numbers $N_{R,L}^{(k)}$ take the fractional value which is a multiple of $1/N$. Alternatively we can express the contribution from sixteen-dimensional gauge part in terms of the Kac-Moody algebra in the following way. Let $\prod_a G_a$ be the gauge group. If the state with the (shifted) momentum $\tilde{P}^{\hat{I}}$ transforms in the representation $\otimes_a R_a$, then we have a formula

$$\frac{1}{2} \sum_{\hat{I}=1}^{16} (\tilde{P}^{\hat{I}})^2 = \sum_a h_a(R_a) \equiv \sum_a \frac{C_2(R_a)}{k_a + C_2(G_a)} , \quad (14)$$

where k_a is a Kac-Moody level and $C_2(R_a)$ ($C_2(G_a)$) is a Casimir for the R_a (adjoint) representation of the group G_a . If the group G_a is Abelian, the conformal dimension of the state carrying its $U(1)$ charge Q_a is given by $h_a(Q_a) = Q_a^2/k_a$.

The physical states of orbifold models should be invariant under the full action of the orbifold twist. For the untwisted sector, this leads to modulo integer conditions

$$P^{\hat{I}} V^{\hat{I}} - p^t v^t = P^{\hat{I}} a^{\hat{I}} = 0 , \quad (15)$$

from which we can identify gauge groups and massless untwisted matter contents. Massless gauge bosons correspond to the $SO(8)$ weights $p_v^t = (0^3; \pm 1)$ and so satisfy $P^{\hat{I}} V^{\hat{I}} = P^{\hat{I}} a^{\hat{I}} = 0$ while massless untwisted matter fields to $p_v^t = (1, 0^2; 0)$. For instance, models with the quasi-standard embedding (8) always contain an E_6 gauge group. Generically, a $U(1)$ factor gauge group appears corresponding to the non-vanishing element of the shift or Wilson lines although some combination of non-vanishing elements may correspond to the Cartan part of a non-Abelian group. When a $U(1)$ corresponds to a basis vector $V_Q^{\hat{I}}$, the level k_Q and the charge Q of the state with a momentum $\tilde{P}^{\hat{I}}$ are given, respectively, by[37, 8]

$$k_Q = 2 \sum_{\hat{I}=1}^{16} (V_Q^{\hat{I}})^2 , \quad (16)$$

² In our convention, we simply remove the last component of the $SO(9, 1)$ momentum to get the transverse $SO(8)$ momentum.

$$Q = \sum_{\hat{I}=1}^{16} V_Q^{\hat{I}} \tilde{P}^{\hat{I}} . \quad (17)$$

The physical states in twisted sectors are singled out by the generalized GSO projection operator[31, 38, 33]

$$G_k \equiv \frac{1}{N} \sum_{h=0}^{N-1} (\Delta_k)^h \quad (18)$$

so that only the states with $\Delta_k = 1$ survive. Here we recall from Ref. [33] the expression for the operator Δ_k in the absence of a Wilson line:³

$$\Delta_k \equiv e^{i\gamma} e^{2\pi i(N_R+N_L)} e^{2\pi i\Theta_k} , \quad (19)$$

$$\Theta_k \equiv \frac{k}{2} \left[\sum_i (v^i)^2 - \sum_{\hat{I}} (V^{\hat{I}})^2 \right] + \left[\sum_{\hat{I}} \tilde{P}^{\hat{I}} V^{\hat{I}} - \sum_i \tilde{p}^i v^i \right] , \quad (20)$$

where the first term in Θ_k , which is important for non-standard embedding cases, expresses Z_N -transformation property of twisted ground states while the second expresses that of vertex operators. The phase $e^{i\gamma}$ described above should be kept in the case of the higher twisted sectors of non-prime order orbifolds. For the case of $Z_{N=3,7}$ orbifold models, the GSO projection is trivial and the level matching condition $m_R^2 = m_L^2$ is known to be enough to guarantee the modular invariance.

3 Constraints on “Anomalous” $U(1)$

String models without left-moving world-sheet supersymmetry, *i.e.*, (0,2) models, often lead to anomalous $U(1)$ symmetry. [Illustrative examples are given in appendix A.] We wish to have a criterion for the appearance of anomalous $U(1)$. In this section, we examine the massless conditions described in the previous section and find that the appearance of anomalous $U(1)$ is constrained by several reasons.

3.1 Visible-Hidden Sector Mixing in Twisted Sector

The first category of such constraints comes from the universal nature of Green-Schwarz mechanism. Suppose that we have a gauge group $U(1)_A \times \prod_a G_a$ and the $U(1)_A$ is anomalous. In the presence of anomalous $U(1)$, the Kähler potential

³ The expression in the presence of Wilson lines can be found in Refs. [39, 33].

of the dilaton-axion chiral multiplet S is given at one-string loop⁴ by[4]

$$K = -\ln \left(S + S^\dagger - \delta_{\text{GS}} V_A \right) , \quad (21)$$

where V_A is the vector multiplet of $U(1)_A$ and δ_{GS} is a constant related to the mixed gravitational anomaly; it is $\text{Tr } Q_A = 96\pi^2 \sqrt{k_A} \delta_{\text{GS}}$ where Q_A and k_A are the charge and level of $U(1)_A$, respectively. On the other hand, the gauge kinetic function f_a of a factor group G_a is given at string tree level by $f_a = k_a S$,

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4} \sum_a k_a \int d^2\theta S W^{\alpha(a)} W_\alpha^{(a)} + \text{H.c.} , \quad (22)$$

where the summation is taken over all gauge groups including the anomalous $U(1)_A$. The existence of the axion-like coupling of $\text{Im } S$ in eq. (22) enables us to cancel the pure $U(1)_A^3$ and mixed $U(1)_A - G_a^2$ anomalies as well as the mixed gravitational one by combining the nonlinear transformation of the dilaton-axion field with the $U(1)_A$ gauge transformation

$$V_A \longrightarrow V_A + \frac{i}{2} (\Lambda - \Lambda^\dagger) , \quad (23)$$

$$S \longrightarrow S + \frac{i}{2} \delta_{\text{GS}} \Lambda , \quad (24)$$

where Λ is a parameter chiral superfield. In any modular invariant string theory in four-dimensions, all the $U(1)$ anomalies should be canceled in this manner[42]. Hence their anomaly coefficients should satisfy the following universality relation:

$$\frac{1}{k_a} \text{Tr}_{G_a} T(R) Q_A = \frac{1}{3} \text{Tr } Q_A^3 = \frac{1}{24} \text{Tr } Q_A \left(\equiv 8\pi^2 \delta_{\text{GS}} \right) , \quad (25)$$

where $2T(R)$ is the index of the representation R , and we have rescaled Q_A so that $k_A = 1$. We refer to this relation as the universal GS relation.

It is important to realize that when the gauge group contains several $U(1)$'s, each $U(1)$ should satisfy the universal GS relation. This property, which can explicitly be confirmed by examples given in Appendices, follows from the uniqueness of the anomalous $U(1)$; we can always find a unique $U(1)_A$ which may be anomalous so that other $U(1)$'s are anomaly free[42, 8]. Then the universal GS relation for this $U(1)_A$ reads

$$\frac{1}{k_a} \text{Tr}_{G_a} T(R) Q_A = \text{Tr } Q_B^2 Q_A = \frac{1}{3} \text{Tr } Q_A^3 = \frac{1}{24} \text{Tr } Q_A = 8\pi^2 \delta_{\text{GS}} , \quad (26)$$

⁴ This Kähler potential includes the term which becomes the Fayet-Iliopoulos term after the dilaton develops the VEV. Notice that the nonlinear transformation (24) explains why we can have the Fayet-Iliopoulos term in supergravity Lagrangian *without* $U(1)$ R symmetry unlike the conventional wisdom in supergravity[40]. We also note that any other one-string loop effects are not considered here including the possible kinetic mixing between several $U(1)$'s[41].

where Q_B is the charge of any non-anomalous $U(1)$ and has been rescaled so that $U(1)_B$ has level one. By using this equation, we can show that any linear combinations, $U(1)_\alpha = \alpha U(1)_A + \beta U(1)_B$ and $U(1)_\beta = \beta U(1)_A - \alpha U(1)_B$ with $\alpha^2 + \beta^2 = 1$, satisfy the same relations as eq. (26) with rescaled GS constants, $\alpha\delta_{\text{GS}}$ and $\beta\delta_{\text{GS}}$, respectively. We thus see that any $U(1)$ satisfies the universal GS relation even if it does not coincide with the true anomalous $U(1)$. This fact is useful in the following discussion.

We can derive several constraints on anomalous $U(1)$ from the universal nature of anomaly in string theory. The basic observation is that if a $U(1)$ symmetry has no mixed $U(1) - G_a^2$ anomaly for a certain group G_a , then all the $U(1) - G_b^2$ anomalies should vanish for any G_b . Therefore we can judge whether a $U(1)$ is anomalous or not by examining just a single type of anomaly. Practically it is easiest to examine the mixed $U(1)$ anomaly with the largest gauge group G_ℓ since the massless conditions tightly restrict the appearance of nontrivial representations of G_ℓ . For instance, massless matter fields in nontrivial representations of E_8 are forbidden and anomalies involving E_8 always vanish. This explains why all (2,2) models which lead to an unbroken E_8 can not have an anomalous $U(1)$ symmetry.

To derive further constraints, we write a gauge group in the form

$$G = G_{\text{vis}} \times G_{\text{hid}} = \left[\prod_a G_a \times U(1)^m \right] \times \left[\prod_b G'_b \times U(1)^{m'} \right], \quad (27)$$

where the visible- and hidden-sector groups G_{vis} and G_{hid} are originated from E_8 and E'_8 , respectively. Generally it is possible that some massless states transform nontrivially under both of visible and hidden groups. If the model contains no such state, we call it the model with the complete separation of the visible and hidden sectors. Then the universal nature (26) clearly tells us that models with the complete separation have no anomalous $U(1)$ symmetry. Actually we have even stronger constraints: A $U(1)_a$ gauge group in the visible sector is anomaly free if there is a hidden group G'_b so that any massless G'_b -charged field has vanishing $U(1)_a$ charge. Notice, furthermore, that there is no mixing between visible and hidden sectors in the untwisted sector: all massless untwisted fields in the visible sector have vanishing E'_8 -momentum $P^{I'} = 0$ and are neutral under G_{hid} and vice versa. The mixing can arise only through the shift (10) of $E_8 \times E'_8$ momenta. Hence we can restrict ourselves to twisted sectors and conclude that *if the visible-hidden sector mixing is absent for massless twisted matter fields, the model has no anomalous $U(1)$ symmetry.*

We now apply the above constraint to show that many models whose gauge group contains an E_7 or E_6 do not have an anomalous $U(1)$. First consider models with an E_7 . Here we restrict ourselves to the models with $k_a = 1$. The expression (14) tells us that the massless condition (13) forbids the representation with the conformal dimension larger than 1. Then, other than the singlet,

massless matter fields can only belong to the representation $\underline{56}$, which has the conformal dimension $h(\underline{56}) = 3/4$ as is seen from $C_2(E_7) = 18$ and $C_2(\underline{56}) = 57/4$. Actually there are many models in which massless $\underline{56}$'s are forbidden in twisted sectors or have vanishing $U(1)$ charges. An example is the T_1 -sector of Z_3 orbifold models in which $c^{(1)} = 1/3$ and thus the field in $\underline{56}$ can not satisfy the massless condition (13). Another example is the T_2 -sector of Z_4 orbifold models in which $c^{(2)} = 1/4$. Even if a massless $\underline{56}$ appears in this sector, it can not have a non-vanishing charge for any $U(1)$ group since the massless condition is already saturated, $h(\underline{56}) + c^{(2)} - 1 = 0$. The same is true in any twisted sectors of Z_3 , Z_4 , Z_6 -I, Z_7 and Z_8 -I orbifold models in the classification of Refs. [32, 33]. Therefore, if the hidden gauge group contains an E_7 , all $U(1)$ symmetries in the visible sector are anomaly free for the Z_N orbifold models other than Z_6 -II, Z_8 -II, Z_{12} -I and -II.

Note, however, that since our constraint here utilizes the absence of the visible-hidden sector mixing, it does not exclude the anomalous $U(1)$ in the same sector as an E_7 . The possible hidden gauge groups which include an E_7 is $E_7' \times U(1)'$ other than anomaly free $E_7' \times SU(2)'$. The $U(1)'$ accompanied by the E_7' is anomalous if the untwisted sector⁵ contains a massless $\underline{56}$. Actually an explicit analysis shows that it *is* always anomalous before Wilson lines are turned on.

A similar analysis can be extended to the models with an E_6 gauge group. The E_6 group with $k = 1$ allows only the $\underline{27}$ and its conjugate representations since we have $C_2(E_6) = 12$, $C_2(\underline{27}) = 26/3$ and $h(\underline{27}) = 2/3$. Therefore the same result as in the models with an E_7 can be derived for the Z_3 orbifold models containing an E_6 gauge group. On the other hand, other Z_N orbifold models are less constrained even if they contain an E_6 gauge group (although the appearance of massless $\underline{27}$'s is not so often since their conformal dimension is large). Here we only make some comments on the models with the quasi-standard embeddings (8). Even if we restrict ourselves to this class of models, there are some examples in which an anomalous $U(1)$ appears even in the opposite sector to the E_6 . If we restrict further to the cases without a Wilson line, however, an explicit analysis shows that the E_6 group derived from the quasi-standard embeddings does not have any nontrivial twisted matter fields and all $U(1)$'s in the opposite sector to the E_6 are anomaly free. Especially in the case of Z_7 orbifold models, all the quasi-standard embeddings do not lead to the anomalous $U(1)$ in the visible as well as hidden sectors.

One may wonder whether the above constraints were not so powerful since the realistic model building involves the Wilson lines which do not allow such a large gauge group as an E_7 or E_6 . As we shall see in the next section, however, the origin of anomalous $U(1)$ can be traced back to the cases without a Wilson line and so the constraints given here turn out to be already powerful.

⁵ In the untwisted sector with $P^{\hat{I}}V^{\hat{I}} \equiv 1/2$ which is $N = 2$ subsector, the degeneracy factor is important.

3.2 Discrete Symmetries

The second category of constraints comes from discrete symmetries of a spectrum. In this section, we examine the mass formulas and describe several examples of such symmetries that forbid the appearance of an anomalous $U(1)$. As is clear from the discussion on $U(1)$ basis given around eq. (26), we can examine the visible and hidden sectors separately and so we concentrate on the visible-sector gauge group coming from the first E_8 .

As noted above, a $U(1)$ gauge symmetry generically corresponds to the non-vanishing element of the shift V^I or Wilson lines a^I ; such non-vanishing element breaks the E_8 and generically corresponds to an unbroken $U(1)$ basis. On the other hand, a vanishing element of the shift and Wilson line generically corresponds to the Cartan basis of a non-Abelian gauge group. Only exception[7, 8] is the case with the total shift of the form $kV^I + m^i a_i^I = (*^7, 0)$, where $*$ indicates non-zero entries. In any case, if J -th and K -th components of the shift and Wilson lines vanish simultaneously, they correspond to the Cartan parts of a non-Abelian group.

In this subsection, we are mainly interested in the $U(1)$'s that correspond to non-vanishing elements of a Wilson line. It is instructive, however, to reconsider why the anomaly cancels for the $U(1)$ which corresponds to a vanishing element of the shift and Wilson line. Suppose that the J -th components of the shift and Wilson line are zero: $V^J = a^J = 0$. Then we observe that $\tilde{P}^J = P^J$ and that the mass formulas (13), physical state conditions (15) and generalized GSO projection operator (19), (20) are all invariant under the transformation which reverses the sign of the J -th component of E_8 -momenta:

$$P^I = (\dots, P^J, \dots) \longrightarrow \bar{P}^I \equiv (\dots, -P^J, \dots) . \quad (28)$$

This implies that if the state with P^I is massless and physical, there exists the massless physical state with \bar{P}^I and thus the $U(1)$ charges corresponding to the J -th Cartan generator sum up to vanish

$$\sum P^J = 0 , \quad (29)$$

where the summation is taken over all massless states. In this way, the absence of the anomaly for the $U(1)$'s corresponding to vanishing elements of the shift and Wilson line can be understood on the basis of the discrete symmetry of the spectrum.

In fact, there is a subtlety in the above discussion. If the E_8 -momentum P^I corresponds to a spinorial root (3), the operation (28) flips the chirality. Then we need another component $V^K = a^K = 0$ so that the simultaneous change $P^J \rightarrow -P^J$ and $P^K \rightarrow -P^K$ preserves the chirality. Otherwise, we have to examine whether or not the states with a spinorial root exist and contribute to the anomaly.

Now let us extend the above argument to show that there is no anomaly in the $U(1)$ corresponding to a non-vanishing element of Wilson lines if the shift and Wilson lines are orthogonal to each other. [This includes the case with a vanishing shift.] We mainly consider the Z_3 orbifold models with a single independent Wilson line a^I for simplicity. Because of the orthogonality, we can use an E_8 transformation so that $V^J = 0$ for $a^J \neq 0$.

We examine the untwisted and twisted sectors separately. For the untwisted sector, precisely the same argument as above applies. If an E_8 -momentum P^J satisfies the massless conditions (15) for untwisted fields, the momentum \bar{P}^I whose J -th component is replaced with $-P^J$ also satisfies it. This is the symmetry from which we conclude that the charges of the $U(1)$ corresponding to the non-vanishing element $a^J \neq 0$ of the Wilson line sums up to vanish in the untwisted sector. For the twisted sector, the momenta of states are shifted as in eq. (10). On each two-dimensional Z_3 orbifold, there are three subsectors corresponding to three fixed points with $e^i = m e_{SU(3)}^i$ ($m = 0, \pm 1$) in eq. (4), where $e_{SU(3)}^i$ is the simple root of the $SU(3)$ lattice. In the subsector corresponding to $m = 0$, the massless condition (13) is symmetric under $P^J \rightarrow -P^J$ and the situation is similar to the untwisted sector. Interestingly, if P^J satisfies the massless condition (13) for $m = 1$, $-P^J$ satisfies it for $m = -1$ and therefore the contributions to the anomaly cancel between $m = 1$ and $m = -1$ subsectors.

Note that if there exists the massless state with a spinorial root, we must take care of the chirality as noted before. We need another component $V^K = 0$ so that the simultaneous change $P^J \rightarrow -P^J$ and $P^K \rightarrow -P^K$ preserves the chirality. This is always possible in Z_3 orbifold models and therefore we have $\sum \tilde{P}^J = \sum \tilde{P}^K = 0$.

We thus see that if the Wilson line is orthogonal to the shift, there exists a discrete symmetry which guarantees the cancellation of anomaly for the $U(1)$ basis that corresponds to non-vanishing elements of the Wilson line. In this case, the anomalous $U(1)$ basis is related only with non-vanishing elements of the shift. *If the E_8 breaking by the shift does not produce an anomalous $U(1)$, there is no anomaly even if several $U(1)$'s appear by switching on the Wilson lines orthogonal to the shift.*

For other orbifold models, we can find similar symmetries. For instance, the two-dimensional Z_4 orbifold has two subsectors corresponding to $m = 0, 1$. The corresponding Wilson line a^I satisfies $2a^I = 0$ up to Γ_{E_8} as was shown in Refs. [39, 33]. This implies that the subsector with $m = 1$ is equivalent to one with $m = -1$ as far as the massless condition is concerned. If there is no massless state with a spinorial \tilde{P}^I , this equivalence will lead to a symmetry of the massless spectrum.

If a Wilson line is not orthogonal to the shift, the symmetry described above is broken. Even in such a case, however, it happens that a similar symmetry is found if the Wilson line and shift satisfy a certain relation. Let us take the

	<u>27</u>		<u>1</u>	
	Q_1	Q_2	$Q_1/\sqrt{3}$	$Q_2/\sqrt{3}$
U_1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	-1	0
U_2	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
U_4	0	-1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

Table 1: $U(1)$ charges of untwisted matters in Z_7 standard embedding

Z_3 orbifold model with $V^J = 2/3$ and $a^J = 2/3$ as an example. The twisted states in the subsector with $m = -1$ have $\tilde{P}^J = P^J$ while the states in the subsectors with $m = 0$ and $m = 1$ have the shifted momenta, $\tilde{P}^J = P^J + 2/3$ and $\tilde{P}^J = P^J + 4/3 = (P^J + 2) - 2/3$, respectively. Therefore the sum of the quantum number $\tilde{P}^{\hat{J}}$ vanishes for the twisted states as in the above discussion (although such a cancellation does not always work in the untwisted sector). A similar cancellation between twisted states can be found in several combinations of the shift and Wilson line.

Next we study another symmetry in the massless spectrum. Consider the Z_7 orbifold model with the standard embedding $V^{\hat{I}} = (1, 2, -3, 0^5; 0^8)/7$. This model has the gauge group $G = E_6 \times U(1)_1 \times U(1)_2 \times E'_8$ whose $U(1)$ bases can be taken as

$$V_1^I = \frac{\sqrt{3}}{2} (1, -1, 0, 0^5) , \quad V_2^I = \frac{1}{2} (1, 1, -2, 0^5) , \quad (30)$$

or equivalently,

$$\tilde{V}_1^I = \frac{1}{2\sqrt{7}} (5, -4, -1, 0^5) , \quad \tilde{V}_2^I = \frac{\sqrt{3}}{2\sqrt{7}} (1, 2, -3, 0^5) . \quad (31)$$

Although the absence of $U(1)$ anomaly in this model can be proved by other reasons such as the existence of the E'_8 or $(2,2)$ superconformal symmetry, this model has remarkable symmetries that guarantee the anomaly cancellation.

The untwisted sector has three subsectors U_1 , U_2 and U_4 which are classified by the value $\sum_i p^i v^i = 1/7, 2/7$ and $4/7$. Each untwisted subsector has a $\underline{27} + \underline{1}$ as the massless matter. In each twisted sector $T_{1,2,4}$, we have

$$7 \times [\underline{27} + \underline{1} + 2 \times \underline{1} + 4 \times \underline{1}] , \quad (32)$$

where the first $\underline{27}$'s come from non-oscillated states, and remaining singlets from oscillated states with $N_L = 1/7, 2/7, 4/7$, respectively. The $U(1)$ charges of these fields are shown in Table 1 for the untwisted states in the basis (30) and in Table 2 for the twisted states in the basis (31). The states in each column of both tables

	$\underline{27} \ (7 \times 1)$		$\underline{1} \ (7 \times 1)$		$\underline{1} \ (7 \times 2)$		$\underline{1} \ (7 \times 4)$	
	$\sqrt{7}\tilde{Q}_1$	$\sqrt{7}\tilde{Q}_2$	$\frac{\sqrt{7}}{2\sqrt{3}}\tilde{Q}_1$	$\frac{\sqrt{7}}{2\sqrt{3}}\tilde{Q}_2$	$\frac{\sqrt{7}}{3}\tilde{Q}_1$	$\frac{\sqrt{7}}{3}\tilde{Q}_2$	$\frac{\sqrt{7}}{\sqrt{3}}\tilde{Q}_1$	$\frac{\sqrt{7}}{\sqrt{3}}\tilde{Q}_2$
T_1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0	1
T_2	$-\frac{1}{2}$	$+\frac{\sqrt{3}}{2}$	0	1	-1	0	$+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
T_4	1	0	$+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

Table 2: $U(1)$ charges of twisted matters in Z_7 standard embedding

form a triplet and thus the anomaly cancels between them. Actually the massless spectrum of this model is symmetric under

$$X^1 \longrightarrow X^2 \longrightarrow X^3 \longrightarrow X^1, \quad (33)$$

which respectively rotates the untwisted subsectors $U_{1,2,4}$ as well as the twisted sectors $T_{1,2,4}$ with the same oscillator number into each other. We can find a similar symmetry in some other orbifold models (even with non-standard embeddings). The partial list includes the untwisted states for Z_8 -I and Z_{12} -I orbifold models with quasi-standard embeddings, whose defining shifts are $v^i = 1/8(1, 2, -3)$ and $1/12(1, 4, -5)$, respectively.

We also note that the above Z_7 orbifold model has another type of symmetry. We see from Table 2 that the $U(1)$ charges of the singlets with different oscillator numbers sum up to vanish in a single twisted sector. [The numbers in parentheses of Table 2 show the multiplicity of the states.] This is the symmetry within each subsector corresponding to the fixed point and might be interesting when we include a Wilson line, whose role is to resolve the degeneracy between the fixed points.

4 Classification of “Anomalous” $U(1)$ in Orbifold Models

In this section we examine in detail Z_3 orbifold models and give a procedure for classifying the models with an anomalous $U(1)$. We work out the models in the absence of a Wilson line and extend the analysis to the case with a Wilson line. Although we deal only with Z_3 orbifold models, our procedure is general and can be applied to other models.

Let us first recapitulate the classification of Z_3 orbifold models in the absence of a Wilson line with attention to the visible-hidden sector mixing. Modular invariant pairs of shifts $(V^I; V^{I'})$ are classified into five types including a trivial one as

$$\text{No. 0 : } (3V^I; 3V^{I'}) = (0^8; 0^8),$$

$$\begin{aligned}
\text{No. 1 : } (3V^I; 3V^{I'}) &= (2, 1, 1, 0^5; 0^8) , \\
\text{No. 2 : } (3V^I; 3V^{I'}) &= (2, 1, 1, 0^5; 2, 1, 1, 0^5) , \\
\text{No. 3 : } (3V^I; 3V^{I'}) &= (1, 1, 0^6; 2, 0^7) , \\
\text{No. 4 : } (3V^I; 3V^{I'}) &= (2, 1^4, 0^3; 2, 0^7) ,
\end{aligned} \tag{34}$$

up to $E_8 \times E'_8$ automorphisms⁶. The first model is a trivial one with unbroken $E_8 \times E'_8$ gauge group and no massless matter field. The second one (model No. 1) corresponds to the standard embedding with $E_6 \times SU(3) \times E'_8$ gauge group. The massless twisted matter fields are

$$(27, 1; 1') \quad \text{for } N_L = 0 , \quad (1, 3; 1') \quad \text{for } N_L = \frac{1}{3} . \tag{35}$$

This model has no visible-hidden sector mixing. The third one (model No. 2) corresponds to a quasi-standard embedding and leads to the gauge group $E_6 \times SU(3) \times E'_6 \times SU(3)'$. This model has the visible-hidden sector mixing due to the massless twisted matter

$$(1, 3; 1', 3') \quad \text{for } N_L = 0 , \tag{36}$$

but this mixing does not contribute to anomaly. Thus the models No. 0, 1 and 2 contain no $U(1)$ gauge group and are anomaly free. On the other hand, an anomalous $U(1)$ does arise in the models No. 3 and 4 as we describe in detail in Appendix A.

We now proceed to models in the presence of a Wilson line. Each subsector corresponding to the fixed point labeled by the intergers m^i has the total shift of the form

$$(V^I + m^i a_i^I; V^{I'} + m^i a_i^{I'}) . \tag{37}$$

It is remarkable that *any modular invariant pair of the total shift is equivalent to one of five shifts (34) up to $E_8 \times E'_8$ automorphisms*. This property enables us to have a simple classification of anomalous $U(1)$. We call the shift $V^{\hat{I}} = (V^I; V^{I'})$ which is equivalent to the total shift (37) of the subsector under consideration as *an equivalent shift*. Similarly we call such a model without a Wilson line that has the equivalent shift of the subsector under consideration as *an equivalent model*. All the complication comes from the fact that such equivalent shifts of subsectors can be different from each other: For example it is possible that the subsector labeled by $m^i = (0, 0, 0)$ has an equivalent shift of No. 0 type while the equivalent shift for $m^i = (1, 0, 0)$ is of No. 1 type.

For an illustrative purpose, let us first discuss the model with the shift No. 1 and a single Wilson line:

$$\begin{aligned}
3V^{\hat{I}} &= (2, 1, 1, 0, 0, 0, 0^2)(0, 0, 0, 0^5)' , \\
3a_1^{\hat{I}} &= (0, 0, 0, 2, 1, 1, 0^2)(2, 1, 1, 0^5)' .
\end{aligned} \tag{38}$$

⁶ One can subtract $E_8 \times E'_8$ roots so that the total shift has the length less than one. See also Ref. [43].

The gauge group $E_6 \times SU(3)_1 \times E'_8$ of the model No. 1 is broken by this Wilson line to

$$SU(3)_1 \times SU(3)_2 \times SU(3)_3 \times SU(2) \times U(1) \times E'_6 \times SU(3)' . \quad (39)$$

The twisted sector of this model has three subsectors corresponding to $m^1 = 0, \pm 1$. The subsector with $m^1 = 0$ has the same structure as the twisted sector of the model No. 1; the massless matter fields appear in the representation of $E_6 \times SU(3)_1 \times E'_8$ as in eq. (35). Actually the E_6 is broken to $SU(3)_2 \times SU(3)_3 \times SU(2) \times U(1)$ under which the $\underline{27}$ decomposes into $(3, 3, 2)_1 + (3, 3, 1)_{-2}$. Nevertheless, it is important to realize that matter fields appear as if they form an E_6 multiplet. It is then clear that this subsector does not contribute to anomaly since the $U(1)$ in eq. (39) is a part of the E_6 in this subsector. Similarly, the subsector with $m^1 = 1$ has the equivalent shift of No. 2 type and the massless matter fields appear in the representation of $E_6 \times SU(3)_1 \times E'_6 \times SU(3)'$. In this case, all the fields are singlet under the broken E_6 and so has vanishing $U(1)$ charges. Thus this subsector does not contribute to anomaly in spite of the visible-hidden sector mixing due to the field (36). [Even if the $SU(3)$ is broken to $SU(2) \times U(1)$ or $U(1)^2$ by another Wilson line, the field $(1, 3; 1', 3')$ does not produce any anomalies.] The subsector with $m^1 = -1$ is also of No. 2 type and has no contribution to anomaly. Thus this model has only No. 1 and 2 types of twisted massless matter fields and does not contain the anomalous $U(1)$.

An important lesson from the above example is that *the massless matter content in any subsector is precisely the same as in the corresponding twisted sector of the equivalent model*. This can directly be checked by looking at the massless condition. [Degeneracy factors also coincide with each other when counted in the corresponding subsector in the equivalent model.] As far as each subsector is concerned, the effect of Wilson lines is just to decompose the representation of matter fields according to the unbroken gauge group, and the situation is quite similar to the conventional Higgs mechanism of grand unified theories. This is not true for a whole model, of course, since each subsector can correspond to a different type of models. Note also that some of untwisted matter fields are projected out by including a Wilson line.

Now it is clear what criteria we have for the appearance of an anomalous $U(1)$: If the total shift (37) is equivalent to one of the shifts No. 0, 1 and 2 listed in eq. (34) for all the subsectors $m^i = 0, \pm 1$, then the model contains no anomalous $U(1)$. On the other hand, a model contains an anomalous $U(1)$ if there is a subsector whose total shift is equivalent to the shift No. 3 or 4. If there are several such subsectors, the anomalous $U(1)$ is their linear combination (as long as the cancellation between them does not occur).

Let us illustrate how our procedure for finding an anomalous $U(1)$ works in concrete examples. First consider the model with

$$3V^{\hat{I}} = (2, 1, 1, 0^5)(2, 1, 1, 0^5)' ,$$

$$3a_1^{\hat{I}} = (-2, 0, 0, 0^5)(0, -1, -1, 0^5)' . \quad (40)$$

This model has the gauge group

$$SO(10) \times SU(2) \times U(1)_1 \times U(1)_2 \times SO(10)' \times SU(2)' \times U(1)'_1 \times U(1)'_2 ,$$

whose $U(1)$ bases we take as

$$\begin{aligned} U(1)_1 : Q_1 &= (0, 1, 1, 0^5)(0^8)' , & U(1)'_1 : Q'_1 &= (0^8)(0, 1, 1, 0^5)' , \\ U(1)_2 : Q_2 &= (1, 0, 0, 0^5)(0^8)' , & U(1)'_2 : Q'_2 &= (0^8)(1, 0, 0, 0^5)' . \end{aligned} \quad (41)$$

The subsectors with $m^1 = 0$ and $m^1 = -1$ have the total shifts equivalent to No. 2 type shift and do not contribute to anomaly. The equivalent shift in the subsector with $m^1 = 1$ is of No. 3 type and thus an anomalous $U(1)$ arises from this subsector. The massless matter fields appear in the representation of broken $E_7 \times U(1)_1 \times SO(14)' \times U(1)'_2$, and as is shown in Appendix A, the $U(1)_1$ is anomalous while the $U(1)'_2$ is anomaly free. The remaining $U(1)_2$ and $U(1)'_1$ are anomaly free since they are contained in the E_7 or $SO(14)'$. In this way, one can conclude that this model contains an anomalous $U(1)$, whose basis is given by $(0, 1, 1, 0^5)(0^8)'$.

Next we discuss the model in which two subsectors contribute to anomaly. An example of such model is given by

$$\begin{aligned} 3V^{\hat{I}} &= (0, 1, 1, 0^5)(2, 0, 0, 0^5)' , \\ 3a_1^{\hat{I}} &= (-2, -1, -1, 0^5)(-2, -1, -1, 0^5)' . \end{aligned} \quad (42)$$

This model has the same gauge group as the previous one and we take the same $U(1)$ bases (41). The subsector with $m^1 = 0$ has the equivalent shift of No. 3 type and the massless matter fields form the multiplets under the broken $E_7 \times U(1)_1 \times SO(14)' \times U(1)'_2$. In particular, there is the visible-hidden mixing due to $(1; 14')_{(2,-1)/3}$, which contributes to anomaly so that $\text{Tr}_{SO(10)'} Q_1 \neq 0$. In addition, the subsector with $m^1 = 1$ also has the total shift equivalent to No. 3 type shift. The massless matter fields appear in the representations under the broken $SO(14) \times U(1)_2 \times E'_7 \times U(1)'_1$ in this subsector. In particular, the existence of $(14; 1')_{(1,-2)/3}$ causes $\text{Tr}_{SO(10)} Q'_1 \neq 0$. The subsector with $m^1 = -1$ is of No. 2 type and does not contribute to anomaly. As a result, the true anomalous $U(1)$ is a linear combination of $U(1)_1$ and $U(1)'_1$, whose coefficients are determined by calculating $\text{Tr}_{SO(10)'} Q'_1$ and $\text{Tr}_{SO(10)} Q_1$ as $Q_1 - Q'_1$.

The third example is the model in which the anomaly cancels between two subsectors:

$$3V^{\hat{I}} = 3a_1^{\hat{I}} = (2, 1, 1, 1, 1, 0^3)(2, 0^7)' . \quad (43)$$

The gauge group is $SU(9) \times SO(14)' \times U(1)'$ as in the model No. 4, but the inclusion of the Wilson line removes all the massless untwisted matter. The

subsector with $m^1 = -1$ has the model No. 0 as an equivalent model and contains no massless matter fields. The matter content of $m^1 = 0$ subsector is the same as in the twisted sector of the model No. 4 and is given by nine copies of $(9, 1')_{2/3}$. In the $m^1 = +1$ subsector, the total shift is $V^{\hat{I}} + a_1^{\hat{I}} = 2V^{\hat{I}}$ which is just the shift of the T_2 -sector in the model No. 4. Therefore this subsector contains nine copies of $(9^*, 1')_{-2/3}$ as is seen from the general fact[44] that the matter content in the T_2 -sector is CPT conjugate of that in the T_1 -sector. These fields cancel the anomaly arising from the $m = 0$ subsector.

We can extend these analysis to more general cases as well as other orbifold models. Actually such an analysis proceeds as follows:

- (i) Classify all the models before a Wilson line is included: In each model, work out its twisted matter content and identify the basis of an anomalous $U(1)$ and its charges.
- (ii) Turn on Wilson lines. For each subsector of a twisted sector T_k ,
 - (a) identify the equivalent model⁷. This subsector has no contribution to anomaly if the T_k -sector of the equivalent model does not contain the massless matter which contributes to the visible-hidden sector mixing.
 - (b) “pull back” the anomalous $U(1)$ basis $V_{Q^{(0)}}$ in the equivalent model to get the basis V_Q of the anomalous $U(1)$ in this subsector. Schematically,

$$\begin{array}{ccc}
\text{shift in } E_8 \times E'_8 \text{ lattice : } & kV^{\hat{I}} + m^i a_i^{\hat{I}} & \xrightarrow{R_{E_8 \times E'_8}} kV^{\hat{I}} \\
& & \downarrow \text{step (i)} \\
\text{anomalous } U(1) \text{ basis : } & V_Q^{\hat{I}} & \xleftarrow{R_{E_8 \times E'_8}^{-1}} V_{Q^{(0)}}^{\hat{I}}
\end{array} \tag{44}$$

- (c) decompose the matter content of the T_k -sector of the equivalent model according to the unbroken gauge group in the presence of Wilson lines. In general, the “pull back” operation as in (44) is necessary also in this step.
- (iii) Put these subsectors as well as the untwisted sector together to get the whole model⁸. The true basis of the anomalous $U(1)$ is given by a linear combination of $U(1)$ bases obtained in the step (b). The massless matter content and $U(1)$ charges are obtained from the results of the step (c).

⁷ A careful analysis shows that the GSO projection in the presence of a Wilson line results in the same degeneracy factor as in the corresponding twisted subsector of the equivalent model.

⁸ It might be interesting to observe that the construction of an orbifold model in a Wilson line background resembles the construction of a fiber bundle.

A classification of modular invariant pairs of the shifts $(V^I; V^{I'})$ is already available[34, 32] and it is straightforward to identify the basis of $U(1)$ which has the visible-hidden sector mixing contributing to anomaly. As an example, a classification of visible-hidden sector mixing in Z_4 orbifold models is given in Appendix B, where several new features are observed.

We finish this section by two remarks. Firstly, the absence of an anomalous $U(1)$ can be established by examining only twisted sectors in spite of the fact that the untwisted sector may contribute to anomaly. This is a special case of more general phenomena: As in the example (42), the true anomalous $U(1)$ is a linear combination of several $U(1)$'s when several subsectors contribute to the anomaly. In such a case, the true basis of the anomalous $U(1)$ can be determined by calculating the mixed anomaly between the visible and hidden sectors and therefore by examining only twisted sectors. These are the consequences of the universal nature of anomaly in string theory as described in section 3.1. Secondly, a cancellation of anomaly may occur between several subsectors which contribute to the anomaly. We should remark that such a cancellation can be understood, in some cases, by the discrete symmetry described in section 3.2. An example is provided by the model (43). We note also that the absence of anomaly in the model (38) can be understood by the discrete symmetry, *i.e.*, by the orthogonality of the Wilson line to the shift. In this way, the analysis based on the discrete symmetries plays a role complimentary to the analysis in this section.

5 “Anomalous” $U(1)$ Breaking and Discrete Symmetries

The anomalous $U(1)$ gauge symmetry breaks automatically once the dilaton VEV is fixed by yet unknown mechanism. There exists a flat direction along which some scalar fields develop the VEV's to cancel the Fayet-Iliopoulos term. Then the next issue to be discussed is along which flat direction the anomalous $U(1)$ breaking occurs, and what kind of consequences we have after such anomalous $U(1)$ breaking. We defer the discussion on the former issue to future publication and comment briefly on the latter here.

In general discrete symmetries survive breaking of a continuous symmetry. Suppose that the anomalous $U(1)$ symmetry is broken by a VEV of a scalar field with the $U(1)$ charge Nq . If other fields which remain massless after this breaking have charges quantized in units of q , a discrete Z_N subgroup of the original anomalous $U(1)$ is left unbroken. This Z_N symmetry has Z_N anomaly which comes from two sources. As was discussed in Ref. [45] for breaking of an anomaly free $U(1)$, a discrete gauge anomaly arises by integrating the matter fields which gain mass terms through the symmetry breaking. A new contribution arises from the original $U(1)$ anomaly. With Tr' denoting a summation over massless fields

after the anomalous $U(1)$ breaking, the Z_N^3 anomaly can be written as

$$\frac{1}{3}\text{Tr}'Q_A^3 = 8\pi^2\delta_{\text{GS}} + \frac{1}{3}\left(mN + \eta n\frac{N^3}{8}\right), \quad (45)$$

where m and n are some integers and $\eta = 1, 0$ for $N = \text{even, odd}$, respectively. The first term is the contribution from the $U(1)$ anomaly and is proportional to the Green-Schwarz coefficient δ_{GS} . The second term is the contribution from massive fields, for which we have used the formula given in Ref. [45]. Similarly the discrete version of the mixed gravitational anomaly is calculated to be

$$\frac{1}{24}\text{Tr}'Q_A = 8\pi^2\delta_{\text{GS}} + \frac{1}{24}\left(pN + \eta q\frac{N}{2}\right), \quad (46)$$

where p and q are integers. The integers m , n , p and q depend on the flat direction along which the anomalous $U(1)$ breaks. If these Z_N anomalies are to be cancelled by the Green-Schwarz mechanism[46, 47], we should have a relation

$$\frac{1}{3}\text{Tr}'Q_A^3 = \frac{1}{24}\text{Tr}'Q_A \quad \text{mod } N, \quad (47)$$

which leads to the following constraint

$$\frac{8m - p}{24} + \eta\frac{2N^2 - q}{48} = \text{integer}. \quad (48)$$

We can regard this as a constraint on possible flat directions. Note that δ_{GS} cancels out and so this constraint is independent of the original anomaly. We can also calculate mixed Z_N anomalies for gauge symmetries, and have further constraints if these anomaly coefficients for Z_N symmetry are to be universal.

Note that the anomalous $U(1)$ breaking sometimes triggers the further gauge symmetry breaking if the scalar fields which develop the VEV's are charged under other gauge groups G . This is the mechanism which is widely used to reduce the rank of gauge group in semi-realistic string models[6, 7, 8]. In such a case, some linear combination of $U(1)$ and G survives and the discrete symmetry in the above discussion may be taken to be orthogonal to it.

In addition to gauge symmetries, there is another type of symmetries which are broken in the course of the anomalous $U(1)$ breaking. We are interested in discrete R symmetries and let us consider them in Z_3 orbifold models as an example. The Z_3 orbifold is symmetric under the independent Z_3 rotation of each complex plane as

$$X^i \longrightarrow e^{2\pi i \tilde{v}^i} X^i, \quad (49)$$

where $\tilde{v}^i = (n^1, n^2, n^3)/3$ with arbitrary integers $0 \leq n^i < 3$. The right-moving world-sheet supersymmetry requires that the fermionic string coordinates $\psi^i \sim$

e^{iH^i} should be rotated simultaneously, and these rotations are realized by the independent shift of each $H^{i=1,2,3}$ as

$$H^i \longrightarrow H^i + 2\pi\tilde{v}^i, \quad (50)$$

which rotates the space-time supercharge $Q \sim \prod_i e^{iH^i/2}$ as

$$Q \longrightarrow e^{\pi i \sum_i \tilde{v}^i} Q \left(= e^{\frac{\pi i}{3} \sum_i n^i} Q \right). \quad (51)$$

Unless $\sum_i \tilde{v}^i$ is an even integer, these discrete symmetries (49) and (50) do not commute with the space-time supersymmetry and are R symmetries[24, 25].

These R symmetries are generated by the rotation by $e^{2\pi i/3}$ of the i -th complex plane. We call such generating element R_i . The R_i charge of a state can be read off as follows. Massless scalar fields in the untwisted sector, generally denoted by $U_{1,2,3}$, have $SO(8)$ momenta $p^t = (p^i; 0)$ with $p^i = (1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, respectively, and the massless twisted field T has $\tilde{p}^t = (1, 1, 1; 0)/3$. These fields transform under the R_i as

$$R_i : U_j \longrightarrow e^{\frac{2\pi i}{3}\delta_{ij}} U_j, \quad T \longrightarrow e^{\frac{2\pi i}{9}} T, \quad (52)$$

from which we see that each R_i generates Z_{18} symmetry.

The R symmetries (49) and (50) can be accompanied by discrete rotations of the left-moving gauge coordinates. In the fermionic formulation, they are Z_3 rotations of $\lambda^{\hat{I}} \sim e^{i\bar{X}^{\hat{I}}}$, which are equivalently realized by discrete shifts of the bosonic coordinates as

$$\bar{X}^{\hat{I}} \longrightarrow \bar{X}^{\hat{I}} + 2\pi\tilde{V}^{\hat{I}}. \quad (53)$$

Here $\tilde{V}^{\hat{I}} = n^{\hat{I}}/3$ with integers $0 \leq n^{\hat{I}} < 3$ if the \hat{I} -th component of the shift $V^{\hat{I}}$ is non-vanishing. [For vanishing component $V^{\hat{I}} = 0$, we should take $\tilde{V}^{\hat{I}} = 0$ since eq. (53) no longer give a symmetry transformation.] In the (2,2) vacuum, in order to preserve the left-moving world-sheet supersymmetry, we should associate such $E_8 \times E'_8$ rotation that is realized by $\tilde{V}^{\hat{I}} = (\tilde{v}^i, 0^5; 0^8)$ with the discrete rotations of X^i and ψ^i . In (0,2) vacua, however, the discrete rotations (53) are independent and we are free to associate them with eqs. (49) and (50). Note that the discrete symmetries (53) themselves are nothing but discrete parts of gauge symmetries.

These R -symmetries (50) and discrete parts (53) of gauge symmetries are generally broken when the anomalous $U(1)$ breaks along a flat direction. Suppose that the matter field which develops the VEV to cancel the Fayet-Iliopoulos term has $SO(8)$ and $E_8 \times E'_8$ momenta \tilde{p}^i and $\tilde{P}^{\hat{I}}$, respectively. The corresponding vertex operator transforms under eqs. (49), (50) and (53) as⁹

$$V_R V_L \longrightarrow e^{2\pi i(\tilde{P}^{\hat{I}}\tilde{V}^{\hat{I}} - \tilde{p}^i\tilde{v}^i)} V_R V_L. \quad (54)$$

⁹ For an oscillated state, the phase $\exp(2\pi i N_L)$ should be multiplied. Note also that if we are to transform the twist field at the same time, we have the corresponding phase from the twisted ground state. In such a case, the R charge of the state can be read off by the formula similar to the GSO phase (20).

Thus the original R symmetries generated by R_i are generally broken, and the surviving R symmetries are such combinations of R_i and discrete symmetries that satisfy

$$\tilde{P}^{\hat{I}} \tilde{V}^{\hat{I}} - \tilde{p}^i \tilde{v}^i = \text{integer} . \quad (55)$$

If several matter fields develop their VEV's, the shifts \tilde{v}^i and $\tilde{V}^{\hat{I}}$ of the surviving R symmetry should satisfy this relation for each pair of \tilde{p}^i and $\tilde{P}^{\hat{I}}$.

Let us illustrate the situation by taking the Z_3 orbifold model with No. 4 shift, whose flat directions were analyzed in Ref. [6]. This model has the gauge group $SU(9) \times SO(14)' \times U(1)'$ and the massless matter content is shown in Appendix A. First consider the simplest flat direction where the negatively charged field $(1, 14')_{-1}$ in the untwisted sector develops the VEV which cancels the positive Fayet-Iliopoulos term and breaks $SO(14)'$ simultaneously. In this case, the discrete gauge symmetry which survives the breaking is Z_6 under which

$$\phi_{64'} \longrightarrow e^{k\pi i} \phi_{64'} , \quad \phi_9 \longrightarrow e^{\frac{4\pi i}{3}k} \phi_9 , \quad (56)$$

where $k = 0, \dots, 5$. On the other hand, if the field $(1, 14')$ which develops the VEV has $p^i = (1, 0, 0)$, the R symmetries generated by R_1 are broken while the $R_{2,3}$ are left unbroken. Although the R_1 is broken, we can find a new unbroken R -symmetry by combining the discrete part of gauge symmetries so as to satisfy eq. (55), *i.e.*, by combining the symmetry (53) generated by $\tilde{V}^{\hat{I}} = (0^8; 2/3, 0^7)$. Notice that this is exactly the basis for the anomalous $U(1)$, and thus the surviving R symmetry is a combination of the broken R_1 and the broken part of anomalous $U(1)$ gauge symmetry.

This Z_3 orbifold model has another flat direction, where $(9, 1')_{2/3}$ as well as $(1, 14')_{-1}$ develop their VEV's. These $(9, 1')_{2/3}$ fields come from the twisted sector and have $\tilde{p}^i = (1, 1, 1)/3$. Switching on their VEV's breaks the above new R -symmetry associated with the anomalous $U(1)$. Even in this case, however, the unbroken R -symmetry can be found by further combining the center of the $SU(9)$ generated by $\tilde{V}^{\hat{I}} = (1, 1, 1, 1, 2, 0, 0, 0; 0^8)/3$.

The other Z_3 orbifold models also have unbroken R -symmetries after symmetry breaking along flat directions. This analysis can be extended to Z_3 orbifold models with Wilson lines and other orbifold models. Unbroken R -symmetries, in general, contain discrete subgroups of broken gauge symmetries including the anomalous $U(1)$.

6 Conclusion and discussion

We have studied the origin of anomalous $U(1)$ gauge symmetry in the orbifold construction of four-dimensional string models. By utilizing the universal nature of the anomaly in string theory, we have derived several conditions for the absence of $U(1)$ anomaly. We also have found several discrete symmetries which guarantee

the cancellation of anomaly. We have then presented a procedure for classifying the orbifold string models which possess an anomalous $U(1)$ and for identifying the true basis of the anomalous $U(1)$.

We have found several constraints on the anomalous $U(1)$, which may be regarded as the first step to have the complete criteria for the appearance of an anomalous $U(1)$. These constraints are rather independent of the detailed structure of models and therefore enable us to conclude the absence of an anomalous $U(1)$ before going into the analysis of massless spectra. According to them, one can conclude the absence of an anomalous $U(1)$ by the following reasons: (i) the absence of the visible-hidden sector mixing in the twisted sectors (as in the models with a large gauge group), (ii) the existence of discrete symmetries, which can be found by examining the relation between the shift and Wilson lines (such as the orthogonality). The former is powerful enough to allow us to classify the models in the absence of a Wilson line. Such a classification is the basis for more general and detailed analysis in the presence of Wilson lines. On the other hand, the latter plays an important role complementary to such a detailed analysis.

One of the main results of the present paper is to give a general procedure for classifying and identifying the anomalous $U(1)$ in orbifold string models. According to our procedure, the problem is reduced to the classification of models in the absence of a Wilson line. Once we work out this restricted class of models and identify what type of shifts and twisted sectors lead to an anomalous $U(1)$, we can easily extend the analysis to the case with Wilson lines. This greatly simplifies the actual analysis of an anomalous $U(1)$ since we can avoid the tedious calculation of $U(1)$ charges.

Our procedure is based on the fact that an orbifold model is constructed by modular invariant combinations of the twisted subsectors corresponding to fixed points and that any such subsector in the presence of Wilson lines is equivalent by $E_8 \times E'_8$ automorphisms to some twisted sector in the absence of a Wilson line. The actual analysis is quite easy when such an equivalence is realized by a trivial $E_8 \times E'_8$ automorphism as we demonstrated in concrete examples. When nontrivial $E_8 \times E'_8$ automorphisms are needed, the analysis will be somehow involved. The investigation on this point will be given elsewhere.

The fact that an orbifold model in a Wilson line background is constructed by assigning various types of shifts to twisted subsectors and by combining such subsectors in a modular invariant way, might be interesting by itself. We have observed the similarity to the construction of a fiber bundle. More importantly, this fact makes it clear that the origin of an anomalous $U(1)$ can be traced back to the orbifold twist itself which is realized by the shift $V^{\hat{I}}$ in the Abelian embedding adopted here. This may not be surprising since it is this twist operation that gives rise to the chiral structure of the models. [In this respect, it may be interesting to recall that $E_8 \times E_8$ heterotic string theory itself may be regarded as an orbifold in the M-theoretical picture[48]. It might also be interesting to speculate that

any chiral structure could be traced to an orbifold in some sense.]

Several problems are left untouched in the present paper. One of the most important problems is to give a general characterization of the flat directions along which the anomalous $U(1)$ breaks. Such an analysis of the flat directions are indispensable for many applications of the anomalous $U(1)$ to the realistic model building, such as the construction of string models with realistic gauge groups and matter contents, fermion mass matrices[11, 12, 13], supersymmetry breaking[19, 20], and the calculation of soft breaking terms[16, 17, 18]. Our results, the classification of an anomalous $U(1)$ in particular, will be useful for such an analysis of the flat directions. Also helpful will be the results of Ref. [49] where generic flat directions of Z_{2n} orbifold models were worked out (although the breaking of the anomalous $U(1)$ was not taken into account).

In attempts to construct a realistic string model, discrete symmetries including R symmetries are important, for instance, to constrain phenomenologically dangerous couplings. We have discussed discrete symmetries that survive the anomalous $U(1)$ breaking and observed a possible relation to the broken anomalous $U(1)$. A further study on such a remnant of the anomalous $U(1)$ will be desired.

It is a remarkable possibility that the anomalous $U(1)$ gauge symmetry plays an important role in constructing a promising model of supersymmetry breaking[19, 20]. If this is the case, it is very important to search for a simple and concrete example of the string model in which the supersymmetry breaking mechanism of ref. [19] is realized. For this purpose, we note that there are many such orbifold models without a Wilson line that possess an anomalous $U(1)$ and that these models deserve further study even if they are not necessarily realistic by themselves.

Recently much work has been devoted to understand nonperturbative aspects in supersymmetric gauge theories and string theories. In particular, it was pointed out in Ref. [2] that the nonperturbative effects of the form e^{-aS} are constrained by the anomalous $U(1)$ as well as discrete symmetries. [This may be understood by observing that if we use the field e^{-S} , the anomalous $U(1)$ is linearly realized and the Green-Schwarz coefficient δ_{GS} is just its anomalous $U(1)$ charge.] In this sense, the anomalous $U(1)$ gauge symmetry might be related to some of nonperturbative and universal aspects of the theory. It is therefore important to study the anomalous $U(1)$ from the viewpoint of string duality and M-theory.

Our approach here to the anomalous $U(1)$ can be extended to other constructions of four-dimensional string models such as Calabi-Yau compactifications[50] and fermionic constructions[51]. In particular, the absence of the visible-hidden sector mixing will provide one of the criteria for anomaly free models in any construction. On the other hand, a new clue might be obtained by studying orbifold models with the non-Abelian embedding. The fact that the rank of gauge groups can be lowered in such a construction[52] might be related to the gauge symmetry

breaking triggered by the anomalous $U(1)$ breaking.

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A Examples of “Anomalous” $U(1)$

Here we give two examples for the models with the anomalous $U(1)$. They are Z_3 orbifold models with No. 3 and 4 type shifts and without Wilson lines. As we argued in section 4, anomalous $U(1)$ in all the Z_3 orbifold models can essentially be traced to these two models even after Wilson lines are added.

The model with No. 4 type shift $V^{\hat{I}} = (1^4, 2, 0^3)(2, 0^7)'/3$ has the gauge group $SU(9) \times SO(14)' \times U(1)'_A$. $U(1)'_A$ charges are calculated by the formula (17) as $Q'_A = V_A^{\hat{I}} \tilde{P}^{\hat{I}}$, where the $U(1)$ basis is given by $V_A = (0^8)(1, 0^7)'$. Note that with this choice of basis, the level of $U(1)'_A$ is given by the formula (16) as $k_A = 2$. The massless matter fields are given in an obvious notation by

$$\begin{aligned} U\text{-sector} &: 3 \times \left[(84, 1')_0 + (1, 14')_{-1} + (1, 64'_s)_{\frac{1}{2}} \right] , \\ T_1\text{-sector} &: 27 \times \left[(9, 1')_{\frac{2}{3}} \right] . \end{aligned} \quad (57)$$

Observe that the untwisted matter $(84, 1')$ has vanishing $U(1)'_A$ charge while the twisted matter $(9, 1')$ is $U(1)'_A$ -charged, signaling that $U(1)'_A$ is anomalous. It is straightforward to check that we have

$$\text{Tr}_{SU(9)} T(R) Q'_A = \text{Tr}_{SO(14)'} T(R) Q'_A = \frac{1}{6} \text{Tr} Q_A'^3 = \frac{1}{24} \text{Tr} Q'_A = 9 . \quad (58)$$

With the rescaling $Q'_A \rightarrow \sqrt{2} Q'_A$, the universal GS relation (26) is indeed satisfied with $8\pi^2 \sqrt{2} \delta_{\text{GS}} = 9$. As discussed in section 3, this universality, which guarantees the anomaly cancellation by GS mechanism, enables us to search for the anomalous $U(1)$.

The second example is the model with No. 3 type shift: $V^{\hat{I}} = (1^2, 0^6)(2, 0^7)'/3$. The gauge group is $E_7 \times U(1)_A \times SO(14)' \times U(1)'_B$ and $U(1)$ charges are

$$\begin{aligned} Q_A &= \sum_{\hat{I}} V_A^{\hat{I}} \tilde{P}^{\hat{I}} , & V_A^{\hat{I}} &= (1, 1, 0^6) (0, 0^7)' , \\ Q'_B &= \sum_{\hat{I}} V_B^{\hat{I}} \tilde{P}^{\hat{I}} , & V_B^{\hat{I}} &= (0, 0, 0^6) (1, 0^7)' . \end{aligned} \quad (59)$$

The massless matter fields with charge (Q_A, Q'_B) are

$$\begin{aligned} U\text{-sector} &: 3 \times \left[(56, 1')_{1,0} + (1, 1')_{-2,0} + (1, 14')_{0,-1} + (1, 64'_s)_{0,\frac{1}{2}} \right] , \\ T_1\text{-sector} &: 27 \times \left[(1, 14')_{\frac{2}{3}, -\frac{1}{3}} + (1, 1')_{-\frac{4}{3}, \frac{2}{3}} + 3 \times (1, 1')_{\frac{2}{3}, \frac{2}{3}} \right] , \end{aligned} \quad (60)$$

where the last singlets with the degeneracy factor 27×3 come from oscillated states $N_L = 1/3$. As argued in section 3, the hidden $U(1)'_B$ is not anomalous since no 56 appears in the twisted sector and there is no mixing between E_7 and

$U(1)'_B$. On the other hand, the presence of $\underline{14}'$ in the twisted sector signals that the visible $U(1)_A$ is anomalous. Actually we have

$$\text{Tr}_{E_7} T(R)Q_A = \text{Tr}_{SO(14)'} T(R)Q_A = \frac{1}{2}\text{Tr} Q_B'^2 Q_A = \frac{1}{12}\text{Tr} Q_A^3 = \frac{1}{24}\text{Tr} Q_A = 18. \quad (61)$$

Note that in the $U(1)$ basis (59), the levels of $U(1)_A$ and $U(1)'_B$ are $k_A = 4$ and $k_B = 2$, respectively. Then after the rescaling $Q_A \rightarrow 2Q_A$ and $Q'_B \rightarrow \sqrt{2}Q'_B$, we find the universal GS relation of the form (26) with $\delta_{\text{GS}} = 9/8\pi^2$.

B Classification of Z_4 orbifold models

Here we classify Z_4 orbifold models with anomalous $U(1)$. Ten independent Z_4 shifts are shown in Table 3 including the trivial one[34]. Modular invariant pairs of shifts $(V^I; V^{I'})$ of Z_4 orbifold models are classified[53] into twelve independent pairs as shown in Table 4, whose third column shows the allowed combinations of E_8 shift V^I and E'_8 shift $V^{I'}$ in terms of the corresponding numbers of the first column in Table 3. The fourth and fifth columns of Table 4 show such massless matter fields in T_1 and T_2 twisted sectors that give rise to visible-hidden sector mixing and contribute to the anomaly with respect to the largest gauge group. All of these fields correspond to non-oscillated states.

The models No. 8, 9 and 10 have no $U(1)$ gauge group. The models No. 1 and 2 have E'_8 and $E'_7 \times SU(2)'$, respectively as the gauge group in the hidden sector and thus are anomaly free owing to the reason discussed in subsection 3.1. The remaining seven models have anomalous $U(1)$. There appear several new features which are absent in the case of Z_3 orbifold models discussed in section 3. Firstly, the T_2 sector has a conjugate pair of massless matter fields, *i.e.*, R and \bar{R} , but these fields generally have different degeneracies¹⁰. Secondly, as we describe shortly, a linear combination of the visible $U(1)$ and hidden $U(1)'$ corresponds to a true basis of the anomalous $U(1)$ in the model No. 5. On the other hand, only the $U(1)'$ group is anomalous in the model No. 4 as well as No. 11. Thirdly in the model No. 7, the twisted sector T_1 has no contribution to the anomaly despite the fact that the whole model has the anomalous $U(1)$. After a Wilson line is included, therefore, a subsector of the T_1 sector does not contribute to an anomaly even if it has the No. 7 type shift as the equivalent shift. Similarly the T_2 sector has no contribution to the anomaly in the models No. 3, 11 and 12. This is the reason why we need to work out which twisted sector contributes to the anomaly before including a Wilson line.

As an example, we explicitly give the result on the model No. 5, whose gauge group is $SO(12) \times SU(2) \times U(1)_\alpha \times SO(14)' \times U(1)'_\beta$. The levels of $U(1)_\alpha$ and

¹⁰ See Ref. [49], where generic massless spectra in the T_n sector of Z_{2n} orbifold models are classified.

$U(1)'_\beta$ are $k_\alpha = 4$ and $k_\beta = 8$ if we use the basis given in Table 3. The massless matter content is given by

$$\begin{aligned}
U_1 : & \quad 2 \times [(12, 2; 1')_{-1,0} + (32, 1; 1')_{1,0} + (1, 1; 64'_s)_{0,2}] , \\
U_2 : & \quad 1 \times [(32, 2; 1')_{0,0} + (1, 1; 1')_{\pm 2,0} + (1, 1; 14')_{0,\pm 2}] , \\
T_1 : & \quad 16 \times [(12, 1; 1')_{-\frac{1}{2},1} + 2 \times (1, 2; 1')_{\frac{1}{2},1}] , \\
T_2 : & \quad 4 \times [(1, 1; 14')_{-1,0} + (1, 1; 1')_{1,\pm 2}] \\
& \quad + 6 \times [(1, 1; 14')_{\mp 1,0} + (1, 1; 1')_{\pm(1,\pm 2)}] ,
\end{aligned} \tag{62}$$

where the degeneracy factors are determined by the generalized GSO projection. Notice that all fields in the U_2 sector and six copies of the fields in the T_2 sector form $N = 2$ hypermultiplets and therefore do not contribute to anomaly. A straightforward calculation shows that

$$\begin{aligned}
\text{Tr}_{G_a} T(R) Q_\alpha &= \frac{1}{8} \text{Tr} Q_\beta^2 Q_\alpha = \frac{1}{12} \text{Tr} Q_\alpha^3 = \frac{1}{24} \text{Tr} Q_\alpha = -4 , \\
\text{Tr}_{G_a} T(R) Q'_\beta &= \frac{1}{4} \text{Tr} Q_\alpha^2 Q'_\beta = \frac{1}{24} \text{Tr} Q_\beta^3 = \frac{1}{24} \text{Tr} Q'_\beta = 16 ,
\end{aligned} \tag{63}$$

where G_a stands for $SO(12)$, $SU(2)$ or $SO(14)'$. We have explicitly confirmed the claim that the universal GS relation does not depend on the choice of $U(1)$ basis. The true basis of the anomalous $U(1)_A$ is then found to be $Q_A \propto -Q_\alpha + 2Q'_\beta$, which is orthogonal to the anomaly free combination $Q_B \propto 4Q_\alpha + Q'_\beta$.

In Z_4 orbifold models with Wilson lines, each subsector corresponding to the fixed point has the total shift of the form $(kV^I + m^i a_i^I; kV^{I'} + m^i a_i^{I'})$, which are also classified into the above twelve types up to $E_8 \times E'_8$ automorphisms. As was commented in section 4, one subsector can have a different type of the equivalent shift from others. Thus the anomaly in each subsector is completely classified by using the results given here in Table 4, and the true anomalous $U(1)$ basis of the whole model can be obtained as a linear combination of the would-be anomalous $U(1)$ bases of several subsectors. We can extend these study to other orbifold models.

#	Shift ($4V^I$)	Gauge group	$U(1)$ basis
0	(00000000)	E_8	
1	(22000000)	$E_7 \cdot SU_2$	
2	(11000000)	$E_7 \cdot U_1$	(11000000)
3	(21100000)	$E_6 \cdot SU_2 \cdot U_1$	(21100000)
4	(40000000)	SO_{16}	
5	(20000000)	$SO_{14} \cdot U_1$	(20000000)
6	(31000000)	$SO_{12} \cdot SU_2 \cdot U_1$	(1-1000000)
7	(22200000)	$SO_{10} \cdot SU_4$	
8	(31111100)	$SU_8 \cdot SU_2$	
9	(1111111-1)	$SU_8 \cdot U_1$	(1111111-1)

Table 3: Shifts for Z_4 orbifold models

No.	Gauge group	$(V^I; V^{I'})$	T_1	T_2
1	$E_6 \cdot SU_2 \cdot U_1 \cdot E'_8$	3 ; 0		
2	$E_6 \cdot SU_2 \cdot U_1 \cdot E'_7 \cdot SU'_2$	3 ; 1		
3	$SO_{16} \cdot E'_6 \cdot SU'_2 \cdot U'_1$	4 ; 3	$(16; 1', 1')_{3/2}$	
4	$SO_{14} \cdot U_1 \cdot E'_7 \cdot U'_1$	5 ; 2	$(14; 1')_{-1, 1/2}$	$(14; 1')_{0, \pm 1}$
5	$SO_{12} \cdot SU_2 \cdot U_1 \cdot SO'_{14} \cdot U'_1$	6 ; 5	$(12, 1; 1')_{-1/2, 1}$	$(1, 1; 14')_{\pm 1, 0}$
6	$SO_{10} \cdot SU_4 \cdot E'_7 \cdot U'_1$	7 ; 2	$(16, 1; 1')_{1/2}$	$(10, 1; 1')_{\pm 1}$
7	$SO_{10} \cdot SU_4 \cdot SO'_{12} \cdot SU'_2 \cdot U'_1$	7 ; 6		$(10, 1; 1', 1')_{\pm 1}$
8	$SU_8 \cdot SU_2 \cdot E'_8$	8 ; 0		
9	$SU_8 \cdot SU_2 \cdot E'_7 \cdot SU'_2$	8 ; 1		
10	$SU_8 \cdot SU_2 \cdot SO'_{16}$	8 ; 4		
11	$SU_8 \cdot U_1 \cdot E'_6 \cdot SU'_2 \cdot U'_1$	9 ; 3	$(8; 1', 1')_{-1, 3/2}$	
12	$SU_8 \cdot U_1 \cdot SU'_8 \cdot SU'_2$	9 ; 8	$(1; 8', 1')_2$	

Table 4: Visible-hidden sector mixing in Z_4 orbifold models

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